## Brevia

## SHORT NOTE

# Pitfalls and procedures for determining fabric orientations in non-oriented bore core 

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#### Abstract

Given three differently oriented boreholes intersecting the same structural fabric, the absolute orientation of that fabric can be calculated even when the azimuthal orientation of the core is unknown. The true fabric is defined by the common intersection of at least three small circles, each of which represents the cone of possible orientations for a core segment. Potential pitfalls associated with automating this simple concept for the analysis of large data sets on a microcomputer include incorporating the effect of errors associated with measuring the fabric in the core, and the nature of the intersection between pairs of small circles. These pitfalls can be overcome by following a procedure which uses a combination of minimum angular dispersion to select representative solutions, and eigenvector calculations to define the mean fabric and its associated error envelope.


## INTRODUCTION

The occurrence of many mineral deposits is intimately associated with local structural features. To obtain information about the variation of lithology and structure with depth, most mineral exploration programs depend upon the information that can be derived from the detailed logging of drill core. The intersection of an inclined planar fabric and a core will appear as an ellipse whose major axis marks the local fabric dip direction (Fig. 1). The problem of determining the absolute orientation of a fabric is complicated by a lack of information on the azimuthal orientation of each core segment. During recovery, the core may rotate in the core barrel. The orientation of the core segment relative to the geometry of the borehole is only rarely available. Few methods can provide the absolute orientation of the core (Goodman 1976, A. Bite personal communication 1992).

Three pieces of information are commonly available for each segment of core: the azimuth and plunge of the borehole at depth (which are obtained from a postdrilling survey of the open borehole), and the maximum dip of the fabric in the core segment (which can be measured using a simple goniometer). Possible orientations of the true attitude of the fabric define a small circle, centred on the borehole axis, with radius $\phi$ (Fig. 1). If two differently oriented boreholes intersect the same fabric, then points common to the two small circles provide estimates (solution poles) of the absolute fabric orientation. For any two small circles, there may be up to four possible intersections. With only two small circles it is not possible to choose which of these intersections is valid. If three differently oriented boreholes


Fig. 1. Diagrammatic representation of the relationship between the absolute orientation of a fabric and the core-pole angle as defined by the measurement of inclination of the fabric relative to a segment of core.
intersect the same fabric, then for ideal data the three small circles will intersect at one common point, defining the pole to the fabric (Ragan 1985, pp. 297-312).
In practice, natural data sets are rarely ideal. With real data, errors associated with the angular measure-
ments preclude the three small circles intersecting at a single common point. More probably the three small circles will define three close, but separate, intersections. A more practical approach therefore would be to calculate for a given set of data all the intersections of all possible pairs of combinations of small circles. Problems with this approach include: (a) selection of the appropriate solution pole; (b) incorporating errors associated with angular measurements; and (c) determining a best estimate of the fabric orientation.

## PROCEDURE

## Outline

The procedure as outlined in Table 1 contains a number of separate steps each of which addresses the
problems identified above. The minimum requirement for the procedure is three observations of the same fabric from three differently oriented segments of core. Each observation comprises the azimuth and inclination of the borehole, and the angle $(\phi)$ of the fabric relative to the axis of the core. For each pair of observations there are three possible types of small circle intersection: acute; tangential; and none. In Step 1 (solution pole calculation) all possible solution poles defined by the intersection of these small circles are calculated. The next step (solution pole selection) determines which pole from each solution set provides the best estimate of the true orientation of the fabric pole. In Step 3 (error estimate) the effects of errors associated with each orientation measurement are assessed. The final step (statistical analysis) involves the summation of all selected intersections to calculate the mean fabric vector.


## Step 1-solution pole calculation

Both Charlesworth \& Kirby (1981) and Ragan (1985, p. 310) have presented procedures for calculating the intersections of two small circles. Our approach involves defining the equation of each small circle as the intersection of a unit sphere with a plane normal to the borehole axis, whose distance from the centre of the sphere is dependent upon $\phi$. This intersection thus defines a small circle in terms of the azimuth and plunge of the borehole, and the core-pole angle $\phi$. Combining the equations for two small circles yields a quadratic function, which can have zero, one or two solutions. When the borehole plunge minus the core-pole angle $\phi$ has a negative plunge (i.e. the small circle overlaps the primitive), it is necessary to consider the opposite end of the cone of solutions also. This will give rise to another possible zero, one or two solutions, bringing the total number of possible solutions to a maximum of four. For a data set with $N$ samples, for which $M$ have borehole plunge $-\phi<0$, the maximum number of intersections (possible solutions) can be calculated as follows:

$$
\begin{equation*}
\sum_{k=1}^{N-M}((N-M)-k) \times 2+\sum_{k=1}^{M}(N-k) \times 4 \tag{1}
\end{equation*}
$$

of which,

$$
\begin{equation*}
\sum_{k=1}^{N}(N-k) \tag{2}
\end{equation*}
$$

represent the true solution pole, and the rest are spurious.

## Step 2-solution pole selection

To determine the mean orientation of the fabric pole, it is necessary to isolate the valid solutions from the spurious intersections. Ideally, the correct solution can be distinguished from its partner(s) by the fact that it lies
within a cluster of other solutions, while its partner's location is random. One possible selection criterion is to compute for each of the (up to four) intersection points the sum of the angular distance from that point to every other possible solution pole defined by all intersections. The angular distance sum will be a minimum for the point which lies closest to the cluster of data.

## Step 3-error estimation

Errors, if randomly distributed, may affect both the orientation of the borehole axis and the magnitude of the core-pole angle. Together these errors can significantly affect the accuracy with which it is possible to define the correct location of the solution pole. Factors moderating this effect depend upon: (a) the orientation of the two borehole segments relative to one another; and (b) the orientation of each of the boreholes relative to the measured fabric. The ideal condition is where the angular difference between the two boreholes is greater than $\approx 30^{\circ}$, and where the two boreholes are not azimuthally collinear (having azimuths which differ by approximately $0^{\circ}$ or $180^{\circ}$ ). This produces an acute intersection between the two small circles. The location of possible solution poles in this situation are well constrained (Fig. 2a). The reliability of the chosen solution pole can be defined by estimating the error associated with each directional observation. In general, borehole orientations are based on precise surveys. The major source of error is in measuring the fabric inclination. The core-pole angle error ( $\Delta \phi$ ) defines the maximum and minimum possible radii for the intersecting small circles. Calculating the position of all of the intersections between the two small circles, and their maximum and minimum limits, gives nine estimates of the location of the solution pole (Fig. 2a).

A worse case scenario occurs when the two boreholes have similar azimuths. In this situation the small circles will intersect tangentially at the pole to the fabric. Any error (i.e. where the measured $\phi$ is too small, or too


Fig. 2. Diagrammatic representation of calculation of possible solution poles from two boreholes. (a) Acute intersection between two differently oriented boreholes. (b) Intersection of two azimuthally collinear boreholes. where measured values for $\phi_{1}$ and $\phi_{2}$ do not initially define any intersections. Dashed circle is the $\phi+\Delta \phi$ necessary to define a tangential intersection. (c) Intersection of two azimuthally collinear boreholes, where measured values for $\phi_{1}$ and $\phi_{2}$ initially define two intersections. Shaded area defines the envelope of possible solutions.
large), will have a significant effect on the calculated position of the solution pole. If the measurement of $\phi$ is too small, the corresponding small circle will be too small, and may not intersect any other small circle (Table 1). There will apparently be no valid solutions. This can be resolved by increasing the smaller $\phi$ ( $\phi_{1}$ in Fig. 2b, for example) by small increments until there is at least one intersection ( $\phi_{1}+\Delta \phi$ in Fig. 2b), and vice versa for $\phi_{2}$. These tangential fits give estimates of the true orientation of the fabric. The error limits on the orientation of the fabric can be determined by computing all the possible intersections defined by systematically decrementing and incrementing $\phi_{1}$ and $\phi_{2}$ by $\Delta \phi_{1 \text { Max }}$ and $\Delta \phi_{2 \mathrm{Max}}$, respectively (Fig. 2b).

Similarly, the measurement of $\phi$ may be larger than the true $\phi$. This case is more difficult to recognize since there will be at least two solution pole intersections. Superficially it could look like the acute intersection case outlined above. In this case, however, the two calculated solution poles are located on opposite sides of the true fabric pole (Fig. 2c). Each of these solution poles is equally valid even though they may be quite divergent from the true fabric pole. Choosing either one of the two solution poles would introduce a systematic bias into the solution set. It is necessary to determine if this situation is in effect before proceeding to the selection process (Table 1). This pitfall can be dealt with in the same manner as when $\phi$ is too small (Fig. 2c).

If there are still no intersections after systematically incrementing and decrementing $\phi$, then the specimen should be re-examined. If upon re-examination, there are still no intersections this could indicate a change in the orientation of the fabric (Laing 1977).

## Step 4-statistical analysis

Each calculated solution does not describe the exact location of the pole to the fabric, but rather gives one estimate of the true fabric location. Iterating the corepole angles $\phi_{1}$ and $\phi_{2}$ by the expected error $\Delta \phi$ yields for each pair of observations a set of solution poles distributed about the true solution. When the small circles acutely intersect, each set will contain nine solutions (Fig. 2a). For the azimuthally compromised cases, each data set will contain a variable number of points which are widely dispersed (Figs. 2b \& c). The mean orientation for each set of solution poles can be represented by its principal eigenvector. The magnitude of the prin-
cipal eigenvalue will reflect the dispersion and number of intersections within a set. For tangential intersections the error ellipse defined by the eigenvector calculation will be larger than the actual envelope of possible solutions (Figs. $2 \mathrm{~b} \& \mathrm{c}$ ). Since these intersections provide only broad estimates of the possible solution pole, we believe it is better to overestimate, rather than underestimate the error associated with these points. A mean estimate of the orientation of the fabric pole is calculated by the summation of all the observation pair eigenvector solutions (Hext 1963, Woodcock 1977, Leinert 1991). Using the eigenvector method to first define the reliability of a single set of solutions, and then the orientation and error of the overall mean, has the advantage that it can give differential weighting to solutions of varying quality.

Data sets that could be analysed using this procedure include multiple differently oriented, but univectorial boreholes, or alternatively a single borehole which changes its orientation with depth. By solving for grouped depth intervals it may be possible to identify domains in the core which reflect the presence of either folds, faults or unconformities (Laing 1977).

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